NONSTEADY-STATE RADIANT-CONDUCTIVE HEAT EXCHANGE IN A SEMITRANSPARENT MEDIUM WITH PHASE TRANSITION

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Phase transition processes have long been well studied, but with the exception of a few efforts [1-3] their subjects have been opaque materials, so that thermal radiation inside the material was not considered. Study of radiant–conductive transport of thermal energy in semitransparent materials with a phase transition of the first sort is of theoretical and practical interest (for growth of semitransparent crystals, heating of ice layers by solar radiation, etc.).

The present study will consider nonsteady-state radiant–conductive heat exchange upon fusion and solidification of a one-dimensional plane layer of semitransparent material of thickness L, located between two opaque surfaces. The following assumptions are made in the mathematical formulation of the problem: all material in the solid phase is crystalline: upon solidification the liquid phase transforms to the solid in a manner such that a localized separation region between solid and liquid phases develops; the medium in which the phase transition occurs has a definite phase-transition temperature, and the phase transition is accompanied by liberation or absorption of a latent heat of phase transition; convection is absent from the liquid phase; the medium is nonscattering, absorbing, and radiating: local thermodynamic equilibrium occurs within the entire volume of the layer.

The solution of the problem reduces to finding the temperature distribution and position of the phase separation boundary as functions of time. Here and below, the subscript 1 will denote the left-hand side, with 2 denoting the right-hand side of the phase separation boundary. The mathematical formulation will be as follows:

$$\rho c_i(T)\frac{\partial T_i}{\partial x} = \frac{\partial}{\partial x}\left(\lambda_i(T)\frac{\partial T_i}{\partial x}\right) - \frac{\partial E_i(T)}{\partial x}, \quad t > 0,$$
$$i = 1,\ 0 < x < y(t);\ i = 2,\ y(t) < x < L; \tag{1}$$

with condition on the phase boundary:

$$\varkappa\rho\frac{dy}{dt} = \lambda_1\frac{\partial T_1}{\partial x}\bigg|_{y^-} - \lambda_2\frac{\partial T_2}{\partial x}\bigg|_{y^+} + \int\limits_0^\infty \left(E_{2,\nu}(y^+) - E_{1,\nu}(y^-)\right)d\nu; \tag{2}$$

and boundary conditions:

$$\lambda_1\frac{\partial T_1}{\partial x} = \sigma_1(t)T_1 + q_1(t), \quad x = 0,$$
$$\lambda_2\frac{\partial T_2}{\partial x} = -\sigma_2(t)T_2 + q_2(t), \quad x = L; \tag{3}$$

the initial temperature distribution and position of the phase transition front are:

$$T(x,\ 0) = \varphi(x),\ y(0) = y_0,\ 0 < y_0 < L. \tag{4}$$

The phase-transition temperature is constant [$T(y(t),\ t) = T_t$]. Here c_1 and c_2 are specific heats; λ_1 and λ_2 are thermal conductivity coefficients.

$$E_1 = 2\pi\int\limits_0^\infty d\nu\int\limits_0^1 \left(I_\nu^+(\mu_1, x, t) - I_\nu^-(\mu_1, x, t)\right)\mu_1 d\mu_1,$$

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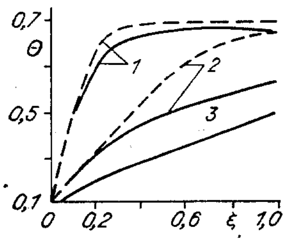

Fig. 1

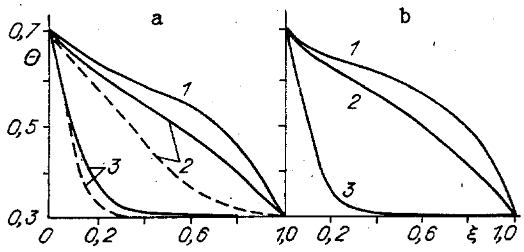

Fig. 2

$$E_2 = 2\pi \int_0^\infty d\nu \int_0^1 \left(I_\nu^+(\mu_2, x, t) - I_\nu^-(\mu_2, x, t)\right) \mu_2 d\mu_2$$

are radiant fluxes in phases 1 and 2; κ is the latent heat of phase transition.

The spectral intensities of radiation forward $I_\nu^+(\mu_i, x, t)$ and to the rear $I_\nu^-(\mu_i, x, t)$ are defined from the radiation transport equations individually for each phase:

$$\begin{aligned} &\mu_i \frac{\partial I_\nu^+}{\partial x} + \alpha_{\nu,i}\left(I_\nu^+ - n_{\nu,i}^2 I_{b,\nu}(T)\right) = 0, \\ &\mu_i \frac{\partial I_\nu^-}{\partial x} - \alpha_{\nu,i}\left(I_\nu^- - n_{\nu,i}^2 I_{b,\nu}(T)\right) = 0 \end{aligned} \tag{5}$$

(with boundary conditions for the intensities from [3]), where i = 1, 2; $\mu_i = |\cos\varphi_i|$; $\alpha_{\nu,i}$ is the spectral absorption coefficient; $n_{\nu,i}$ is the spectral index of refraction; $I_{b,\nu}(T)$ is the spectral intensity of radiation of an ideal black body in a vacuum.

In the present study, Eqs. (1)-(4) were solved by an interpolation method [4, 5], for which the integral conservation laws were satisfied with a difference approximation of the thermal conductivity equation. An implicit difference network was used with the drive method combined with the iteration method. Calculations were halted when temperature fields as calculated for two successive iterations coincided within the specified degree of accuracy. This method permits tracking the motion of the phase separation boundary with a high degree of accuracy. In determining the radiant fluxes integrals of the form

$$K_n = \int_0^1 \mu^{n-2} \exp(-h/\mu)\, d\mu, \quad I(x) = \int_0^x f(x') K_n(x - x')\, dx'$$

were calculated with Gauss quadratures and trapezoid expressions, respectively.

We write Eqs. (1) and (2) in dimensionless form

$$C_i \frac{\partial \Theta_i}{\partial \tau} = N \frac{\partial}{\partial \xi}\left(\Lambda_i \frac{\partial \Theta_i}{\partial \xi}\right) - \frac{\partial \Phi_i}{\partial \xi}, \quad \tau > 0, \tag{6}$$

$$i = 1,\ 0 < \xi < z(\tau);\ i = 2,\ z(\tau) < \xi < 1;$$

$$\frac{Y}{N}\frac{dz}{d\tau} = \Lambda_1 \frac{\partial \Theta_1}{\partial \xi}\Big|_{z^-} - \Lambda_2 \frac{\partial \Theta_2}{\partial \xi}\Big|_{z^+} + \frac{1}{N}\int_0^\infty \left(\Phi_{\omega,2}(z^+) - \Phi_{\omega,1}(z^-)\right) d\omega\,, \tag{7}$$

where $\xi = x/L$; $\tau = \sigma_0 T_r^3 t/\rho c_r L$; $z = y/L$; $\Theta_i = T_i/T_r$; $C_i = c_i/c_r$; $\Lambda_i = \lambda_i/\lambda_r$; $\Phi_{\omega,i} = E_{\omega,i}/\sigma_0 T_r^4$; $Y = \kappa/c_r T_r$; σ_0 is the Stefan–Boltzmann constant; $N = \lambda_r/\sigma_0 T_r^3 L$ is the radiation-conduction parameter; and r is an index defining the parameter.

To test the algorithm, a numerical calculation of T-field formation and motion of the phase boundary was performed for fusion and solidification of a one-dimensional plane layer with initial data from [1], which corresponds approximately to fusion of fluorite [T_t = 1700 °K, $\lambda_2 = \lambda_r$ = 9 W/(m·°K), L = 0.1 m]. All calculations were performed in the gray approximation without consideration of temperature dependence of properties.

The initial data for the fusion problem in dimensionless form are as follows: C_1 = 0.75, C_2 = 1, Λ_1 = 2, Λ_2 = 1, Y = −0.1, with emissivities of both opaque surfaces $\varepsilon_1 = \varepsilon_2$ = 1. Boundary and initial conditions are as follows:

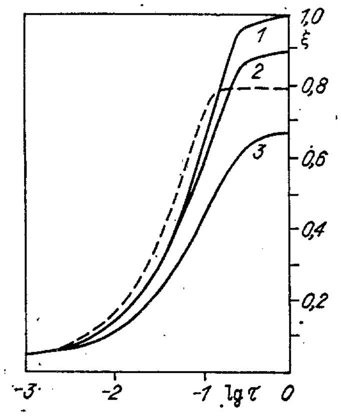

Fig. 3

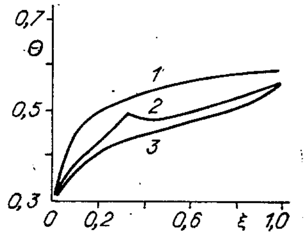

Fig. 4

$$\Theta(0,\tau)=0{,}7,\ \Theta(1,\tau)=0{,}3,\ \Theta(\xi,0)= \\ =0{,}3,\ \Theta(z,\tau)=0{,}5,\ z(0)=0, \\ \tau>0,\ 0<\xi<1.$$

For the solidification problem $C_1 = 1$, $C_2 = 1.25$, $\Lambda_1 = 1$, $\Lambda_2 = 0.5$, $Y = 0.1$. Both boundary surfaces are absolutely black. The boundary and initial conditions are: $\Theta(0, \tau) = 0.3$, $\partial\Theta(1, \tau)/\partial\xi + \sigma_2\Theta(1, \tau) = q_2$, $\sigma_2 = 0.2$, $q_2 = 0.12$, $\Theta(\xi, 0) = 0.7$, $\Theta(z, \tau) = 0.5$, $z(0) = 0$, $\tau > 0$, $0 < \xi < 1$.

We will consider the concrete results of the calculation. Figure 1 shows the temperature-field distribution for solidification, with solid lines corresponding to N = 0.08, and dashed lines to pure thermal conductivity, $n_1 = n_2 = 1.5$, $h_1 = 1$, $h_2 = 2$, $\tau = 0.215$, 2.695, and 12.8 (lines 1-3). Figure 2 shows temperature-field formation during fusion of the speciment with identical indices of refraction (a) and indices differing in the liquid and solid phases (b) [N = 0.05, $h_1 = 1$, $h_2 = 2$; a) $n_1 = n_2 = 1.5$, $\tau = 3.36$, 1.5, 0.11; dashed lines, pure thermal conductivity; b) $n_1 = 1.75$, $n_2 = 1.5$, $\tau = 6.2$, 1.5, 0.11; lines 1-3]. The difference in indices of refraction leads to refraction, reflection, and total internal refraction of the radiation flux on the phase boundary, as a consequence of which motion of the phase-transition front accelerates and exits to a steady-state regime (Fig. 3, dashed line).

Figure 3 shows the time dependence of the position of the phase boundary for various values of N (1, 0.01; 2, 0.05; 3, pure thermal conductivity) during fusion. It is evident that radiation accelerates the phase-transition process.

Calculations reveal that at certain physical parameter values (for small values of N) the classical Stefan condition is violated on the phase separation boundary. This implies that in place of a boundary, a phase-transition region appears, as was noted in [6]. Consequently, the model selected proves inapplicable for calculations. This is illustrated by Fig. 4, where a supercooled region appears on the temperature profile in the vicinity of the phase boundary (N = 0.01, $h_1 = 1$, $h_2 = 2$, $\tau = 2.367$, 5.5, 10; lines 1-3).

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